

Fifth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Modern Control Theory

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

PART - A

- 1 a. Define the following terms:
 i) State ii) State variables iii) State vector iv) State space. (04 Marks)
 b. Obtain the state model for the electrical circuit shown in Fig.Q1(b). (08 Marks)

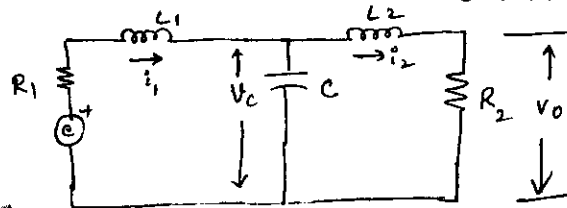


Fig.Q.1(b)

- c. Derive the state model for the system described by the differential equation $D^3y + 4D^2y + 5Dy + 2y = 2D^2u + 6Du + 5u$, where $D = d/dt$, in Jordan canonical form. (08 Marks)

- 2 a. Obtain the state model in first companion form for the system given by $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 4y = 10u$. (06 Marks)

- b. Fig.Q.2(b) shows block diagram of a control system using state variable feedback and integral control. State model of plant is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Derive the state model of entire system. (08 Marks)

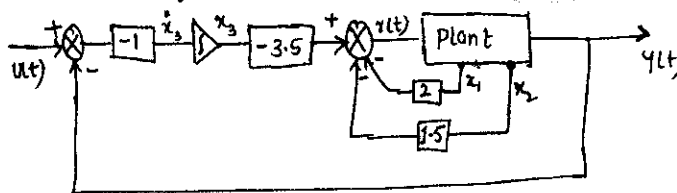


Fig.Q.2(b)

- c. List out atleast one advantage and one disadvantage of selecting,
 i) Physical variables; ii) Phase variable; iii) Canonical variables for state space formulation of control systems. (06 Marks)

- 3 a. Find eigen values, eigen vectors and modal matrix for $A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & 0 \\ 8 & 2 & -5 \end{bmatrix}$. (06 Marks)

- b. Convert the following state model into canonical form by diagonalising matrix 'A'. $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. (08 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. And revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- c. Find the state transition matrix e^{At} for the system using Cayley-Hamilton method

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

(06 Marks)

- 4 a. Find the time response for unit step input of a system given by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) \text{ and } x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(12 Marks)

- b. Evaluate controllability and observability of the following state model,

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & -4 \end{bmatrix} x + \begin{bmatrix} 40 \\ 10 \\ 0 \end{bmatrix} u, y = [0 \ 0 \ 1] x$$

(08 Marks)

PART - B

- 5 a. "Any unstable system can be stabilized by complete state feedback if all the state variables are controllable". Comment on this statement and explain for the Fig.Q.5(a). (06 Marks)

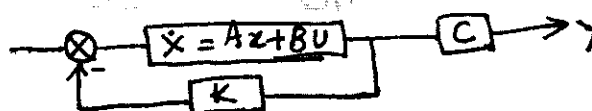


Fig.Q.5(a)

- b. Consider the system defined by $\dot{x} = Ax + Bu$, $y = cx$ where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

By using the state feedback control $u = -kx$, it is desired to have the closed loop poles at $s = -1 \pm j2$ and $s = -10$. Determine the state feedback gain matrix 'K' using Ackermann's formula. (08 Marks)

- c. A regulator system has the plant.

$$\dot{x} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u, y = [0 \ 0 \ 1] x$$

Design an observer such that the eigen values of the observer are located at $-2 \pm j\sqrt{12}$. Use direct substitution method. (06 Marks)

- 6 a. Write a short note on P, PI and PID controllers. (06 Marks)
 b. List the properties of nonlinear systems. (06 Marks)
 c. Draw the input-output characteristics of following nonlinearities and explain in detail:
 i) Dead zone ii) Backlash. (08 Marks)

- 7 a. Explain the phenomenon of jump resonance with respect to the nonlinear systems. (06 Marks)
 b. Find out the singular points for the following: $\ddot{x} + 0.5\dot{x} + 2x = 0$. (04 Marks)
 c. Draw the phase plane trajectory for the following equation using isocline method:

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2x = 0. \text{ Given } \xi = 0.5, \omega_n = 1, \text{ with } x = 0 \text{ and } \frac{dx}{dt} = 1 \text{ as initial condition.}$$

(10 Marks)

Define the following terms:

- i) Positive definiteness.
- ii) Positive semidefiniteness.
- iii) Negative definiteness.
- iv) Negative semidefiniteness.

Give one example to each.

(06 Marks)

- b. Explain the Lyapunov second method and stability theorems. (08 Marks)
 c. Use Krasovskii's theorem to show that the equilibrium state $\mathbf{x} = 0$ of the system described by $\dot{x}_1 = -3x_1 + x_2$; $\dot{x}_2 = x_1 - x_2 - x_2^3$ is asymptotically stable in large. (06 Marks)
